2023
Summer Work for Students Entering Geometry

BelovED
COMMUNITY
CHARTER SCHOOL

Dear Parents and Students:

This packet has been designed to provide a review of Algebra 1 skills that are essential for student success in Geometry. Students should complete this booklet during the summer and **bring it, with work shown, to school on the first day of Geometry**. Students who do so will **start the 2023-2024 school year with a test grade of 100%.** Completion of this packet over the summer before beginning Geometry will be of great value to helping students successfully meet the academic opportunities awaiting them in Geometry and beyond. These opportunities include:

- The Preliminary Scholastic Aptitude Test (PSAT), Scholastic Aptitude Test (SAT) I and II, and American College Test (ACT)
- College placement and advanced placement (AP) tests.

Resources included in this packet are the following:

- A listing of the Algebra 1 objectives that students need to have mastered to be successful in Geometry,
- A series of 10 problem sets consisting of critical content the Algebra 1 curriculum that will help students prepare for Geometry,
- Worked out exemplar problems for students to guide student in each problem set.
- Answer key for all odd numbered problems.

Directions:

- Students are requested to work in pencil and **show their work** in the packet or on lined paper to accompany the packet. They should check their answers using the key provided and, if possible, correct the work for problems solved incorrectly. Round answers to the nearest tenth.
- A calculator may be used on any of the problems in this packet.

Families are encouraged to use the many resources available at the following websites:

- https://www.freemathhelp.com/algebra-help.html has links to dozens of mathematics related websites containing activities, tutorials.
- https://www.khanacademy.org/math/algebra -- offers videos that assist on various algebra topics.
- https://www.mathtutordvd.com/products/item8.cfm?affID=freemath has Algebra videos aligned with content in this packet.

Sincerely,

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A. Simplifying Polynomial Expressions

I. Combining Like Terms

- You can add or subtract terms that are considered "like", or terms that have the same variable(s) with the same exponent(s).

Ex. 1: 5x - 7y + 10x + 3y

5x - 7y + 10x + 3y

15x - 4y

Ex. 2: $-8h^2 + 10h^3 - 12h^2 - 15h^3$

$-8h^2 + 10h^3 - 12h^2 - 15h^3$

$-20h^2 - 5h^3$

II. Applying the Distributive Property

- Every term inside the parentheses is multiplied by the term outside of the parentheses.

$$Ex.\ 1:\ 3(9x-4)$$
$$3\cdot 9x - 3\cdot 4$$
$$27x - 12$$

$$Ex.\ 2:\ 4x^2(5x^3+6x)$$
$$4x^2\cdot 5x^3 + 4x^2\cdot 6x$$
$$20x^5 + 24x^3$$

III. Combining Like Terms AND the Distributive Property (Problems with a Mix!)

- Sometimes problems will require you to distribute AND combine like terms!!

$$Ex.\ 1: 3(4x-2)+13x$$
$$3\cdot 4x - 3\cdot 2 + 13x$$
$$12x - 6 + 13x$$
$$25x - 6$$

$$Ex.\ 2:\ 3(12x-5)-9(-7+10x)$$
$$3\cdot 12x - 3\cdot 5 - 9(-7) - 9(10x)$$
$$36x - 15 + 63 - 90x$$
$$-54x + 48$$

PRACTICE SET 1

Simplify.

1. $8x - 9y + 16x + 12y$

2. $14y + 22 - 15y^2 + 23y$

3. $5n - (3 - 4n)$

4. $-2(11b - 3)$

5. $10q(16x + 11)$

6. $-(5x - 6)$

7. $3(18z - 4w) + 2(10z - 6w)$

8. $(8c + 3) + 12(4c - 10)$

9. $9(6x - 2) - 3(9x^2 - 3)$

10. $-(y - x) + 6(5x + 7)$

B. Solving Equations

I. Solving Two-Step Equations

A couple of hints:

1. To solve an equation, UNDO the order of operations and work in the reverse order.
2. REMEMBER! Addition is "undone" by subtraction, and vice versa. Multiplication is "undone" by division, and vice versa.

$$Ex.1:\ 4x-2=30 \\ +2\quad +2 \\ 4x=32 \\ \div 4 \quad \div 4 \\ x=8$$

$$Ex.2:\ 87=-11x+21 \\ -21 \qquad -21 \\ 66=-11x \\ \div -11 \quad \div -11 \\ -6=x$$

II. Solving Multi-step Equations With Variables on Both Sides of the Equal Sign

- When solving equations with variables on both sides of the equal sign, be sure to get all terms with variables on one side and all the terms without variables on the other side.

$$Ex.3:\ 8x+4=4x+28 \\ -4 \qquad -4 \\ 8x=4x+24 \\ -4x \quad -4x \\ 4x=24 \\ \div 4 \quad \div 4 \\ x=6$$

III. Solving Equations that need to be simplified first

- In some equations, you will need to combine like terms and/or use the distributive property to simplify each side of the equation, and then begin to solve it.

$$Ex.4:\ 5(4x-7)=8x+45+2x \\ 20x-35=10x+45 \\ -10x \qquad -10x \\ 10x-35=45 \\ +35 \quad +35 \\ 10x=80 \\ \div 10 \quad \div 10 \\ x=8$$

PRACTICE SET 2

Solve each equation. You must show all work.

1. $5x - 2 = 33$
2. $140 = 4x + 36$
3. $8(3x - 4) = 196$
4. $45x - 720 + 15x = 60$
5. $132 = 4(12x - 9)$
6. $198 = 154 + 7x - 68$
7. $-131 = -5(3x - 8) + 6x$
8. $-7x - 10 = 18 + 3x$
9. $12x + 8 - 15 = -2(3x - 82)$
10. $-(12x - 6) = 12x + 6$

IV. Solving Literal Equations

- A literal equation is an equation that contains more than one variable.
- You can solve a literal equation for one of the variables by getting that variable by itself (isolating the specified variable).

$Ex.1:\ 3xy = 18,\quad \text{Solve for } x.$

$$\frac{3xy}{3y} = \frac{18}{3y}$$

$$x = \frac{6}{y}$$

$Ex.2:\ 5a - 10b = 20,\quad \text{Solve for } a.$

$$\begin{aligned} +10b &= +10b \\ 5a &= 20 + 10b \\ \frac{5a}{5} &= \frac{20}{5} + \frac{10b}{5} \\ a &= 4 + 2b \end{aligned}$$

PRACTICE SET 3

Solve each equation for the specified variable.

1. $Y + V = W$, for V

2. $9wr = 81$, for w

3. $2d - 3f = 9$, for f

4. $dx + t = 10$, for x

5. $P = (g - 9)180$, for g

6. $4x + y - 5h = 10y + u$, for x

C. Rules of Exponents

Multiplication: Recall $(x^m)(x^n) = x^{(m+n)}$ *Ex*: $(3x^4y^2)(4xy^5) = (3 \cdot 4)(x^4 \cdot x^1)(y^2 \cdot y^5) = 12x^5y^7$

Division: Recall $\frac{x^m}{x^n} = x^{(m-n)}$ *Ex*: $\frac{42m^5j^2}{-3m^3j} = \left(\frac{42}{-3}\right)\left(\frac{m^5}{m^3}\right)\left(\frac{j^2}{j^1}\right) = -14m^2j$

Powers: Recall $(x^m)^n = x^{(m \cdot n)}$ *Ex*: $(-2a^3bc^4)^3 = (-2)^3(a^3)^3(b^1)^3(c^4)^3 = -8a^9b^3c^{12}$

Power of Zero: Recall $x^0 = 1, x \neq 0$ *Ex*: $5x^0y^4 = (5)(1)(y^4) = 5y^4$

PRACTICE SET 4

Simplify each expression.

1. $(c^5)(c)(c^2)$
2. $\frac{m^{15}}{m^3}$
3. $(k^4)^5$
4. d^0
5. $(p^4q^2)(p^7q^5)$
6. $\frac{45y^3z^{10}}{5y^3z}$
7. $(-t^7)^3$
8. $3f^3g^0$
9. $(4h^5k^3)(15k^2h^3)$
10. $\frac{12a^4b^6}{36ab^2c}$
11. $(3m^2n)^4$
12. $(12x^2y)^0$
13. $(-5a^2b)(2ab^2c)(-3b)$
14. $4x(2x^2y)^0$
15. $(3x^4y)(2y^2)^3$

D. Binomial Multiplication

I. Reviewing the Distributive Property

The distributive property is used when you want to multiply a single term by an expression.

Ex 1: $8(5x^2 - 9x)$

$8 \cdot 5x^2 + 8 \cdot (-9x)$

$40x^2 - 72x$

II. Multiplying Binomials – the FOIL method

When multiplying two binomials (an expression with <u>two</u> terms), we use the "FOIL" method. The "FOIL" method uses the distributive property <u>twice</u>!

FOIL is the order in which you will multiply your terms.

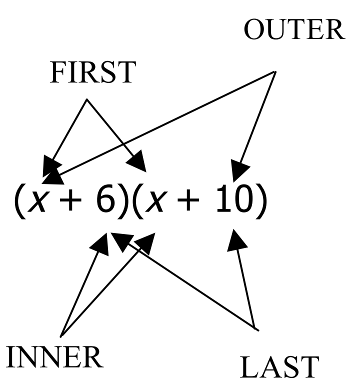
First

Outer

Inner

Last

Ex. 1: (x + 6)(x + 10)



First $x \cdot x$ ------> x^2

Outer $x \cdot 10$ -----> $10x$

Inner $6 \cdot x$ ------> $6x$

Last $6 \cdot 10$ -----> 60

$x^2 + 10x + 6x + 60$

$x^2 + 16x + 60$
(After combining like terms)

Recall: $4^2 = 4 \cdot 4$

$x^2 = x \cdot x$

Ex. $(x + 5)^2$

$(x + 5)^2 = (x + 5)(x+5)$

Now you can use the "FOIL" method to get a simplified expression.

PRACTICE SET 5

Multiply. Write your answer in simplest form.

1. $(x + 10)(x - 9)$
2. $(x + 7)(x - 12)$
3. $(x - 10)(x - 2)$
4. $(x - 8)(x + 81)$
5. $(2x - 1)(4x + 3)$
6. $(-2x + 10)(-9x + 5)$
7. $(-3x - 4)(2x + 4)$
8. $(x + 10)^2$
9. $(-x + 5)^2$
10. $(2x - 3)^2$

E. Factoring

I. Using the Greatest Common Factor (GCF) to Factor.

- Always determine whether there is a greatest common factor (GCF) first.

Ex. 1 $3x^4 - 33x^3 + 90x^2$

- In this example the GCF is $3x^2$.
- So when we factor, we have $3x^2(x^2 - 11x + 30)$.
- Now we need to look at the polynomial remaining in the parentheses. Can this trinomial be factored into two binomials? In order to determine this make a list of all of the factors of 30.

30			30	
1	30		-1	-30
2	15		-2	-15
3	10		-3	-10
5	6		-5	-6

Since -5 + -6 = -11 and (-5)(-6) = 30 we should choose -5 and -6 in order to factor the expression.

- The expression factors into $3x^2(x-5)(x-6)$

Note: Not all expressions will have a GCF. If a trinomial expression does not have a GCF, proceed by trying to factor the trinomial into two binomials.

II. Applying the difference of squares: $a^2 - b^2 = (a-b)(a+b)$

Ex. 2 $4x^3 - 100x$

$4x(x^2 - 25)$

$4x(x-5)(x+5)$

Since x^2 and 25 are perfect squares separated by a subtraction sign, you can apply the difference of two squares formula.

PRACTICE SET 6

Factor each expression.

1. $3x^2 + 6x$
2. $4a^2b^2 - 16ab^3 + 8ab^2c$
3. $x^2 - 25$
4. $n^2 + 8n + 15$
5. $g^2 - 9g + 20$
6. $d^2 + 3d - 28$
7. $z^2 - 7z - 30$
8. $m^2 + 18m + 81$
9. $4y^3 - 36y$
10. $5k^2 + 30k - 135$

F. Radicals

To simplify a radical, we need to find the greatest perfect square factor of the number under the radical sign (the radicand) and then take the square root of that number.

$Ex. 1: \sqrt{72}$

$\sqrt{36} \cdot \sqrt{2}$

$6\sqrt{2}$

$Ex. 2: 4\sqrt{90}$

$4 \cdot \sqrt{9} \cdot \sqrt{10}$

$4 \cdot 3 \cdot \sqrt{10}$

$12\sqrt{10}$

$Ex. 3: \sqrt{48}$

$\sqrt{16}\sqrt{3}$

$4\sqrt{3}$

OR

$Ex. 3: \sqrt{48}$

$\sqrt{4}\sqrt{12}$

$2\sqrt{12}$ ← This is not simplified completely because 12 is divisible by 4 (another perfect square)

$2\sqrt{4}\sqrt{3}$

$2 \cdot 2 \cdot \sqrt{3}$

$4\sqrt{3}$

PRACTICE SET 7

Simplify each radical.

1. $\sqrt{121}$
2. $\sqrt{90}$
3. $\sqrt{175}$
4. $\sqrt{288}$
5. $\sqrt{486}$
6. $2\sqrt{16}$
7. $6\sqrt{500}$
8. $3\sqrt{147}$
9. $8\sqrt{475}$
10. $\sqrt{\dfrac{125}{9}}$

G. Graphing Lines

I. Finding the Slope of the Line that Contains each Pair of Points.

Given two points with coordinates (x_1, y_1) and (x_2, y_2), the formula for the slope, *m*, of the line containing the points is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Ex. (2, 5) and (4, 1)

$$m = \frac{1-5}{4-2} = \frac{-4}{2} = -2$$

The slope is -2.

Ex. (-3, 2) and (2, 3)

$$m = \frac{3-2}{2-(-3)} = \frac{1}{5}$$

The slope is $\frac{1}{5}$

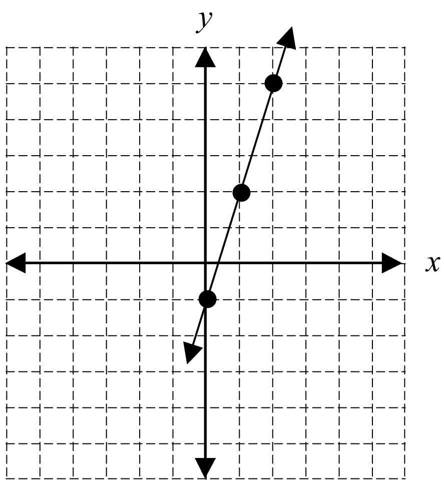
<u>PRACTICE SET 8</u>

1. (-1, 4) and (1, -2)
2. (3, 5) and (-3, 1)
3. (1, -3) and (-1, -2)
4. (2, -4) and (6, -4)
5. (2, 1) and (-2, -3)
6. (5, -2) and (5, 7)

II. Using the Slope – Intercept Form of the Equation of a Line.
The slope-intercept form for the equation of a line with slope m and y-intercept b is $y = mx + b$.

Ex. $y = 3x - 1$

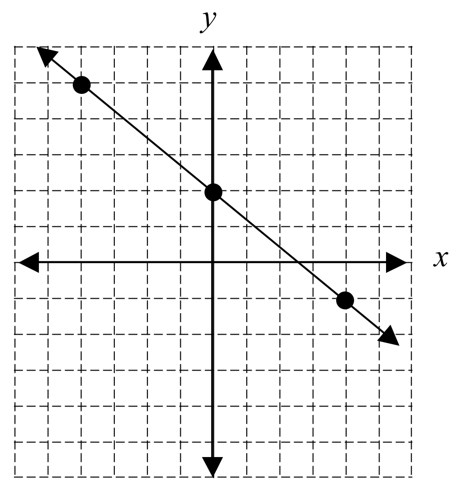
Slope: 3 y-intercept: -1



Place a point on the y-axis at -1.
Slope is 3 or 3/1, so travel up 3 on the y-axis and over 1 to the right.

Ex. $y = -\frac{3}{4}x + 2$

Slope: $-\frac{3}{4}$ y-intercept: 2

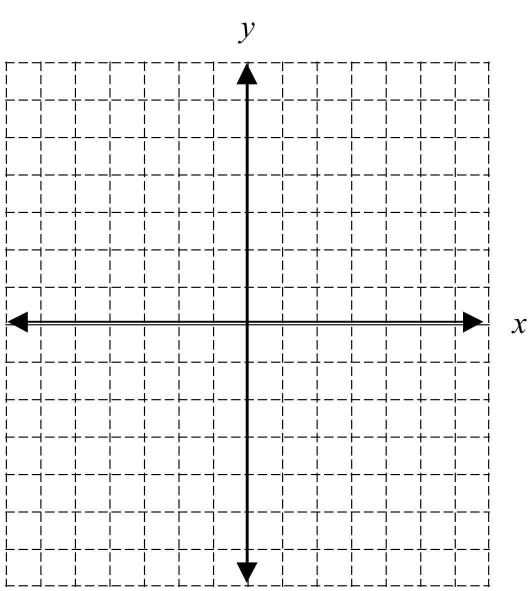


Place a point on the y-axis at 2.
Slope is -3/4 so travel down 3 on the y-axis and over 4 to the right. Or travel up 3 on the y-axis and over 4 to the left.

PRACTICE SET 9

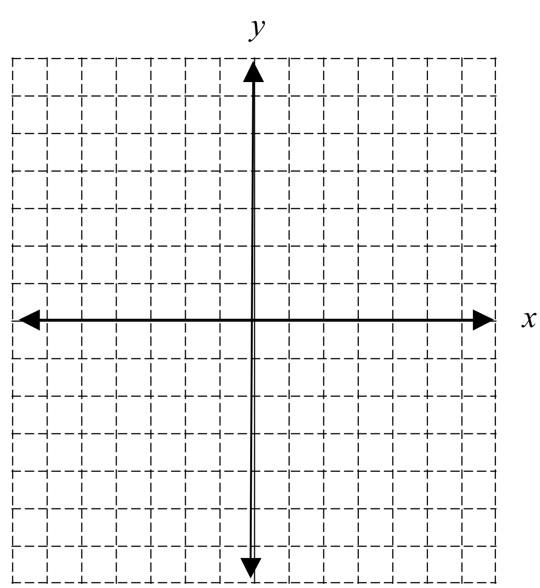
1. $y = 2x + 5$

Slope: _____ y-intercept: _____



2. $y = \frac{1}{2}x - 3$

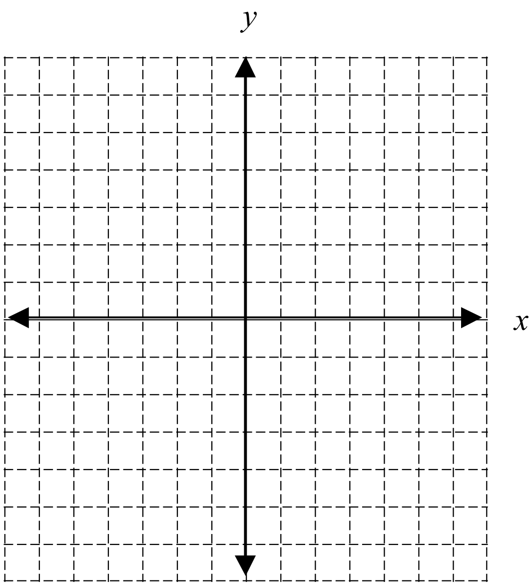
Slope: _____ y-intercept: _____



3. $y = -\frac{2}{5}x + 4$

Slope: _______________

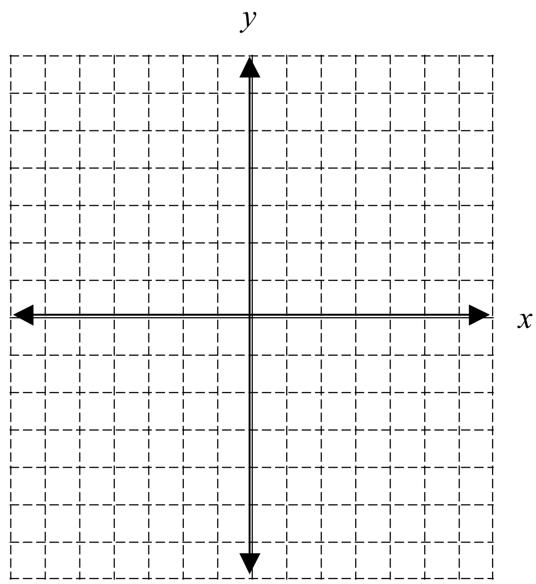
y-intercept: _______________



4. $y = -3x$

Slope: _______________

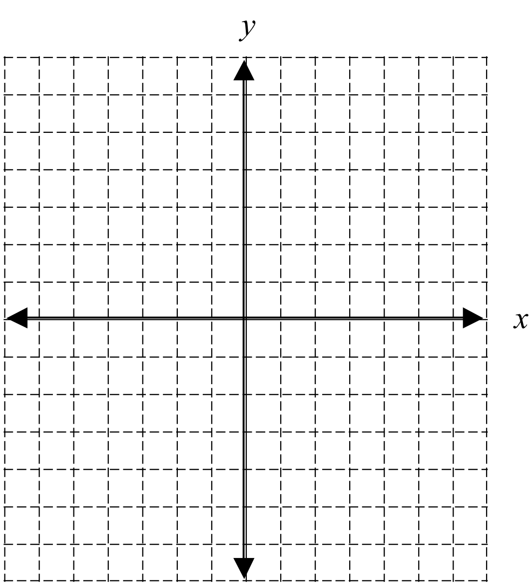
y-intercept _______________



5. $y = -x + 2$

Slope: _______________

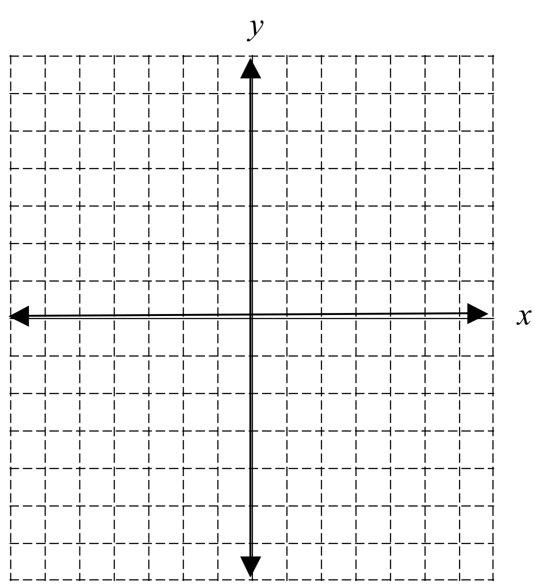
y-intercept: _______________



6. $y = x$

Slope: _______________

y-intercept _______________



III. Using Standard Form to Graph a Line.

An equation in standard form can be graphed using several different methods. Two methods are explained below.

a. Re-write the equation in $y = mx + b$ form, identify the y-intercept and slope, then graph as in Part II above.

b. Solve for the x- and y- intercepts. To find the x-intercept, let $y = 0$ and solve for x. To find the y-intercept, let $x = 0$ and solve for y. Then plot these points on the appropriate axes and connect them with a line.

Ex. $2x - 3y = 10$

a. Solve for y.

$$-3y = -2x + 10$$

$$y = \frac{-2x + 10}{-3}$$

$$y = \frac{2}{3}x - \frac{10}{3}$$

OR

b. Find the intercepts:

let $y = 0$:

$$2x - 3(0) = 10$$

$$2x = 10$$

$$x = 5$$

So x-intercept is (5, 0)

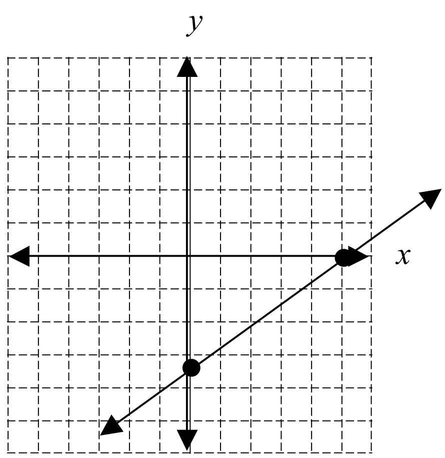
let $x = 0$:

$$2(0) - 3y = 10$$

$$-3y = 10$$

$$y = -\frac{10}{3}$$

So y-intercept is $\left(0, -\frac{10}{3}\right)$



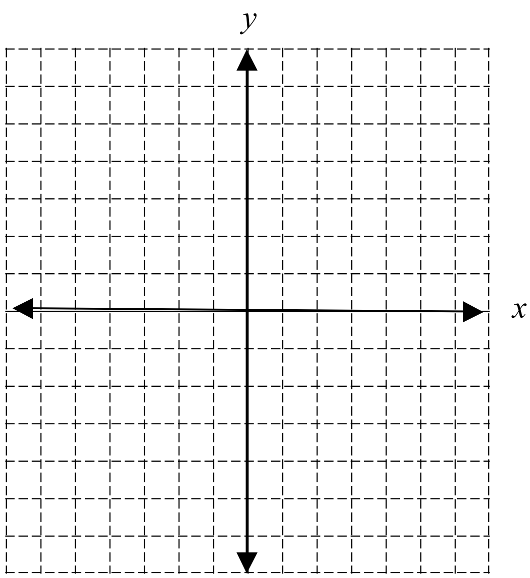
On the x-axis place a point at 5.

On the y-axis place a point at $-\frac{10}{3} = -3\frac{1}{3}$

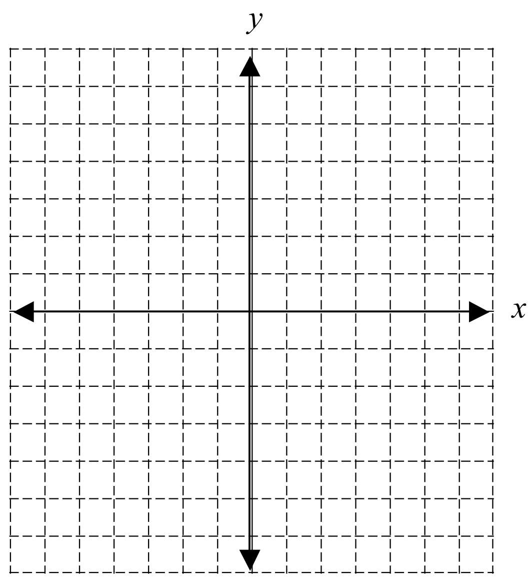
Connect the points with the line.

PRACTICE SET 10

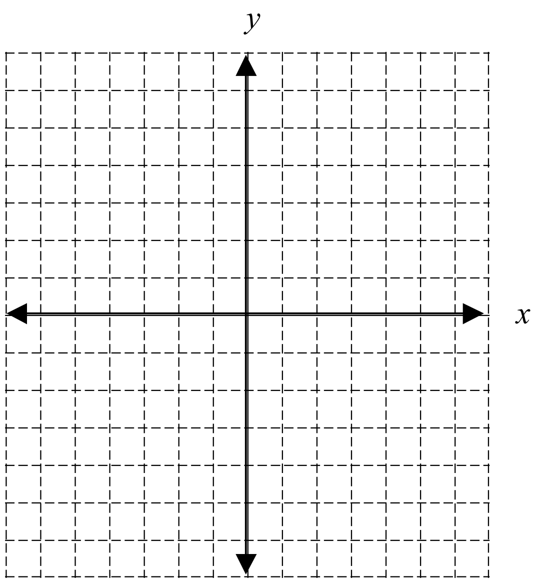
1. $3x + y = 3$



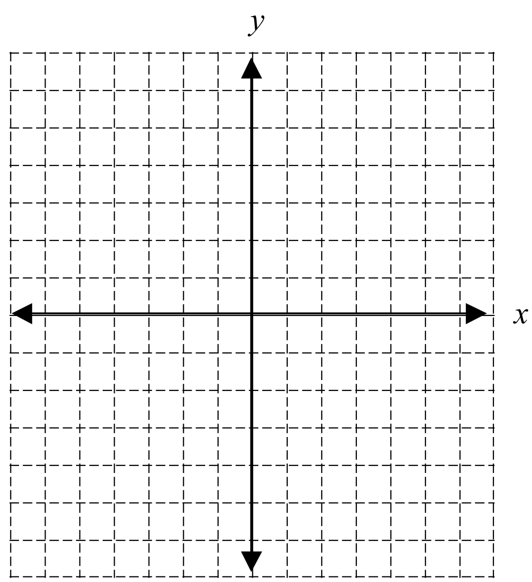
2. $5x + 2y = 10$



3. $y = 4$



4. $4x - 3y = 9$



Algebra 2 Summer Review Packet
Student Answer Key

A. Simplifying Polynomial Expressions

PRACTICE SET 1

1. $24x + 3y$

3. $9n - 3$

5. $160qx + 110q$

7. $74z - 24w$

9. $-27x^2 + 54x\ !\ 9$

B. Solving Equations

PRACTICE SET 2

1. $x = 7$

3. $x = 9.5$

5. $x = 3.5$

7. $x = 19$

9. $x = 9.5$

PRACTICE SET 3

1. $V = W - Y$

3. $f = \frac{9-2d}{-3} = -3 + \frac{2}{3}d$

5. $g = \frac{P+1620}{180} = \frac{P}{180} + 9$

C. Rules of Exponents

PRACTICE SET 4

1. c^8

3. k^{20}

5. $p^{11}q^7$

7. $-t^{21}$

9. $60h^8k^5$

11. $81m^8n^4$

13. $30a^3b^4c$

15. $24x^4y^7$

D. Binomial Multiplication

PRACTICE SET 5

1. $x^2 + x - 90$

3. $x^2 - 12x + 20$

5. $8x^2 + 2x - 3$

7. $-6x^2 - 20x - 16$

9. $25 - 10x + x^2$

E. Factoring

PRACTICE SET 6

1. **$3x(x + 2)$**

3. **$(x - 5)(x + 5)$**

5. **$(g - 4)(g - 5)$**

7. **$(z - 10)(z + 3)$**

9. **$4y(y - 3)(y + 3)$**

F. Radicals

PRACTICE SET 7

1. **11** [redacted] 3. **$5\sqrt{7}$**

5. **$9\sqrt{6}$** [redacted] 7. **$60\sqrt{5}$**

9. **$40\sqrt{19}$**

G. Graphing Lines

PRACTICE SET 8

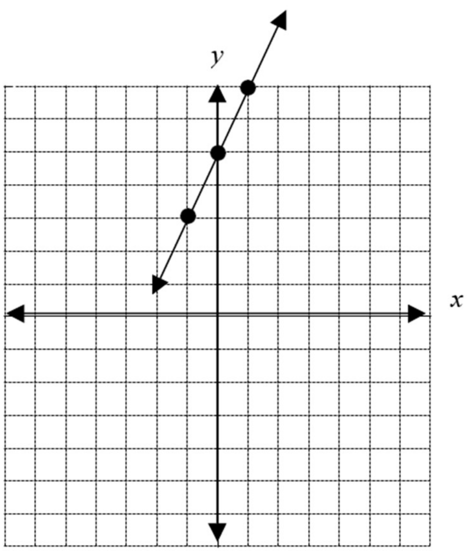
1. **–3**

3. $\frac{1}{2}$

5. **1**

PRACTICE SET 9

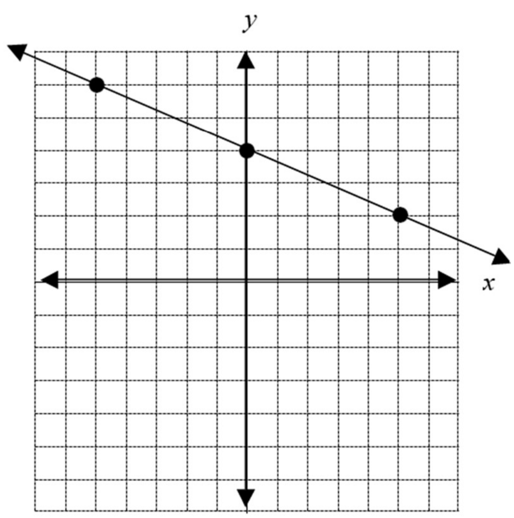
1. Slope: ***2*** y-intercept: **5**



3. $y = -\frac{2}{5}x + 4$

Slope: ____ **$-\frac{2}{5}$** ________

y-intercept: _______**4**_________



5. $y = -x + 2$

Slope: _____**-1**_______

y-intercept: _______**2**_______